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Electron transport in a double quantum dot controlled by magnetic switching

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Abstract

We investigate the possibility of performing single-electron controlled transport in coupled quantum dots based on magnetic switching. From numerical solution of the time-dependent Schrödinger equation it is shown that certain combinations of static and switched magnetic fields can result in a situation where an initially localized wavefunction can be transferred from one of the dot centres to the other one with unit probability.

1. Introduction

The possibility of manipulation of electrons in coupled quantum dots opens a very interesting field of research, both from the point of view of possible applications and as a part of fundamental research in quantum physics. The possible applications range from metrological uses to research in quantum computing. Extensive literature exists on the possibilities and promises of such controlled electron transport [1–7]. Coupled two-dimensional quantum dots, i.e. quantum wells separated by nanometre distances, can be prepared and studied by various experimental techniques. Of particular relevance for the present paper is the recent experiment [8] in which a single electron initially placed in one of the coupled wells is partially transferred to the other well by exposing the system to a train of short external 'top hat' pulses of electric field changes.

The present model is motivated by our experience in the field of atom–atom and ion–atom collisions. In transient quasimolecular systems consisting of the colliding ions or atoms the phenomenon of resonant charge transfer is the model which we aim to implement in the context of quantum wells. In short, the resonant charge transfer is based on existence of two equal energy levels, one in each of the two subsystems when they are isolated, i.e. separated effectively by an infinite barrier. When the barrier is removed or partially removed, the two levels are superimposed and energetically split, as is well known from many textbook examples, e.g. [9], usually known as bonding and antibonding orbitals. When the parameters of the ion–ion collision provide an appropriate energetic splitting of the two 'orbitals', which is also matched by the duration of the collision, a complete transfer can result. In general, however, a partial transfer happens and resonant charge transfer is a very well known process [10].

In the collisions the control of the parameters is generally not possible *a priori*, but the experimental observations with selections of specific types of collision confirmed the basic quantal and semiclassical predictions [10]. In the case of the coupled quantum dots the splitting of the states, the duration of switching process, as well as other similar parameters, might be quite precisely controlled in a carefully designed experiment.

An important question is to what extent and under which conditions the transfer can be complete and coherent. This implies that a wavefunction describing a single unit of charge localized in one of the quantum wells is transferred without dispersion or decoherence to another quantum well. The transfer is induced by a time-dependent electromagnetic field which can be prepared and turned on/off by external devices. Therefore, it is of great importance to achieve a sufficient degree of dynamic control of the quantum electronic system.

In this work we address very similar questions to those considered in the above quoted experiment [8], but we explore numerically whether the well-to-well transfer can be controlled by time-dependent external magnetic fields. Our model is based on a double quantum dot placed in a rather strong magnetic field. The actual switching is accomplished by weakening the magnetic field for a certain short period of time. We consider the initial state as the electron being localized in one of the wells. A change of the field will induce the transfer to the other well. Our proposed switching technique is robust in the sense that the actual shape of the double well potential and the magnetic pulse is not crucial for the efficiency. The challenge is to realize fields of the right intensity, frequency and duration that can cause a complete transfer [6].

In the following a one-electron model of a single electron in a parabolic double quantum dot will be described. Scaled units are applied in this work in order to maintain generality of the results. However, we build our system of units in close relation to the SI system, used in the laboratory, so that the model predictions can be directly related to realistic experimental conditions. For that purpose it is enough to choose a suitable unit for each of the four quantities, i.e. length, mass, time and electric current. Thus, the unit of mass is chosen to be the value of effective electron mass, and the unit of energy is set to the so-called confinement strength $\hbar\omega_0$, which in fact is simply the choice of characteristic electron energy as a unit;

$$\begin{aligned} U_{\text{mass}} &= m^* \\ U_{\text{energy}} &= E_0 = \hbar\omega_0 \\ U_{\text{time}} &= \frac{\hbar}{U_{\text{energy}}} \end{aligned} \tag{1}$$

with ω_0 being the confining trap frequency. Now the unit of length follows from,

$$U_{\text{energy}} = U_{\text{mass}}(U_{\text{length}})^2(U_{\text{time}})^{-2}, \tag{2}$$

i.e.

$$U_{\text{length}} = \left(U_{\text{energy}}U_{\text{time}}^2U_{\text{mass}}^{-1}\right)^{1/2}. \tag{3}$$

Setting the unit of charge equal to the elementary charge, $U_{\text{charge}} = e$, leads to the unit of current,

$$U_{\text{current}} = \frac{e}{U_{\text{time}}}. \tag{4}$$

Following the definitions of the SI units, the unit of magnetic field strength is obtained as,

$$U_{\text{Bfield}} = \frac{U_{\text{mass}}}{U_{\text{time}}U_{\text{charge}}}. \tag{5}$$

Though not used in this paper, we add for completeness the unit of electric field strength,

$$U_{\text{Elfield}} = \frac{U_{\text{mass}}}{U_{\text{time}}^2U_{\text{charge}}}. \tag{6}$$

This somewhat lengthy description has the advantage that the scaling to different parameters is rather obvious. As an example, choosing a larger confinement strength leads to a smaller time unit and thus to a linearly larger magnetic field unit.

In real systems the confinement strength $\hbar\omega_0$ is typically about 1 meV, and the effective mass of an electron in a GaAs material is $m^* = 0.067m_e$, where m_e is the electron mass. With these numbers the unit of time becomes about 0.66 ps, the unit of length about 34 nm, the unit of magnetic field strength about 0.6 T and the unit of electric field strength about 30 kV m^{-1}. We shall refer to this system of units as *adjusted units*, i.e. by the abbreviation au (but not atomic units) and the numerical model will be described with m^*, $\hbar$ and the electron charge e replaced by 1. Two coupled quantum wells are conveniently described by a double harmonic oscillator potential [11, 12],

$$V(x, y; d) = \frac{1}{2}\omega_0^2 \min\left[\left(x - \frac{d}{2}\right)^2 + y^2, \left(x + \frac{d}{2}\right)^2 + y^2\right]. \tag{7}$$

Here, d is the inter-dot distance which separates the two wells. The single-electron Hamiltonian reads,

$$H(x, y, t) = -\tfrac{1}{2}\left(\partial_x^2 + \partial_y^2\right) + V(x, y; d) + V_{\text{ext}}(x, y, t), \tag{8}$$

where $V_{\text{ext}}(x, y, t)$ is the external time-dependent potential,

$$V_{\text{ext}}(x, y, t) = \tfrac{1}{2}w_B^2(x^2 + y^2) + w_B L_z, \tag{9}$$

where $\omega_B = \frac{B}{2}$ is the time-dependent Larmor frequency and $L_z = \mathrm{i}y\partial_x - \mathrm{i}x\partial_y$ is the angular momentum operator. The external potential thus describes a magnetic field parallel with the $\hat{z}$-direction. The spatial extent of this direction is assumed to be so small that this degree of freedom can otherwise be considered frozen. Note that within this model the electron spin is not affected. An additional component of the B-field in, for example, the $\hat{x}$-direction would open for simultaneous manipulation of the electron spin without interference with the spatial dynamics. Such single-electron spin manipulation has been experimentally demonstrated in double quantum dots [13].

The dynamics is governed by the two-dimensional time-dependent Schrödinger equation,

$$\mathrm{i}\partial_t \Psi(x, y, t) = H(x, y, t)\Psi(x, y, t). \tag{10}$$

A numerical solution can be obtained by expanding the wavefunction in a basis set containing a large number of eigenstates of the $d = 0$ au harmonic oscillator,

$$\Psi(x, y, t) = \sum_{n,m} a_{nm}(t)\phi_{n,m}(x, y), \tag{11}$$

which by standard projection techniques leads to a set of coupled differential equations for the amplitude vector $\mathbf{a}(t) = (a_{00}, a_{01}, a_{10} \ldots)$,

$$\mathrm{i}\partial_t \mathbf{a}(t) = \mathbf{H}(t)\mathbf{a}(t). \tag{12}$$

This set of equations is then integrated with the appropriate initial conditions by an efficient and stable numerical method [14].

For sufficiently long inter-dot distances d the two wells couple so weakly that local states in either well are stable eigenstates. The energy spectrum in this limit is very close to the spectrum of two independent harmonic oscillators, at least for the lower lying states. This is also true for the limit $d \to 0$. In the intermediate region these degenerate energy levels are splitting. If the inter-dot distance could be varied with time, similar to the internuclear distance in diatomic molecules, the variation of d would lead to energy splitting and induce dynamics similar to gerade–ungerade charge cloud fluctuations in a diatomic molecule [15]. A

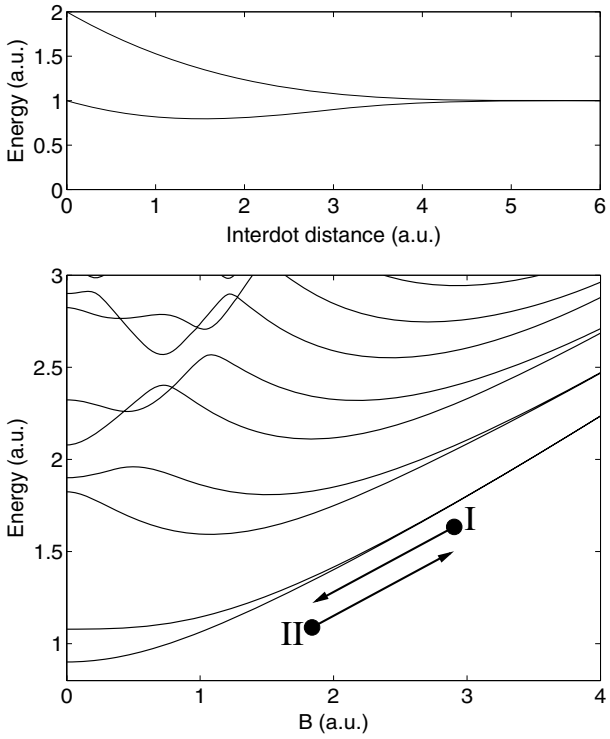

Figure 1. Upper panel: the two lowermost energy levels of the double quantum dot as a function of inter-dot distance d ($\omega_0 = 1$ au). Lower panel: energy levels of the double dot system with $d = 3$ au as a function of the magnetic field strength. The magnetic field points in the $\hat{z}$-direction and the transition arrows indicate the field change process.

sudden decrease of d at a certain time could then lead to a complete charge transfer. This idea is illustrated in the upper panel of figure 1, showing the two lowest energy levels followed from diagonalization of the Hamiltonian equation (8) for different values of the inter-dot distance.

Unlike the transient molecular systems, the studied double quantum dots cannot be changed geometrically, i.e. their distance d is obviously fixed. On the other hand, the barrier or plateau between the two wells can be perturbed by external electromagnetic sources. In our case, the control is assumed to be accomplished entirely by a quasistationary external magnetic field. In the lower panel of figure 1 we display the energy curves of equation (8) for various values of the magnetic field strength and for fixed d. A number of crossings and avoided crossings are seen. The splitting between the two lowest energy levels is seen to decrease with increasing field strength. Thus, the magnetic field strength can play the same role as a virtual variation of the inter-dot distance: a localized initial electron in a strong magnetic field will be allowed to couple to the other quantum dot if the field strength is suddenly reduced. We will now explore this possibility.

We start with an electron localized in the left well. The initial state is a linear combination of a *gerade* and an *ungerade* molecular state, $\Psi_{\mathrm{L}} = \frac{1}{\sqrt{2}}(\psi_{\mathrm{g}} + \psi_{\mathrm{u}})$. The magnetic field is initially so strong ($B = 3$ au) that the initial state remains quasistationary for times several orders of magnitude longer than the actual switching time. The field is then suddenly switched down to about $B = 1.85$ au, and the charge cloud starts tunnelling between the dot centres. After

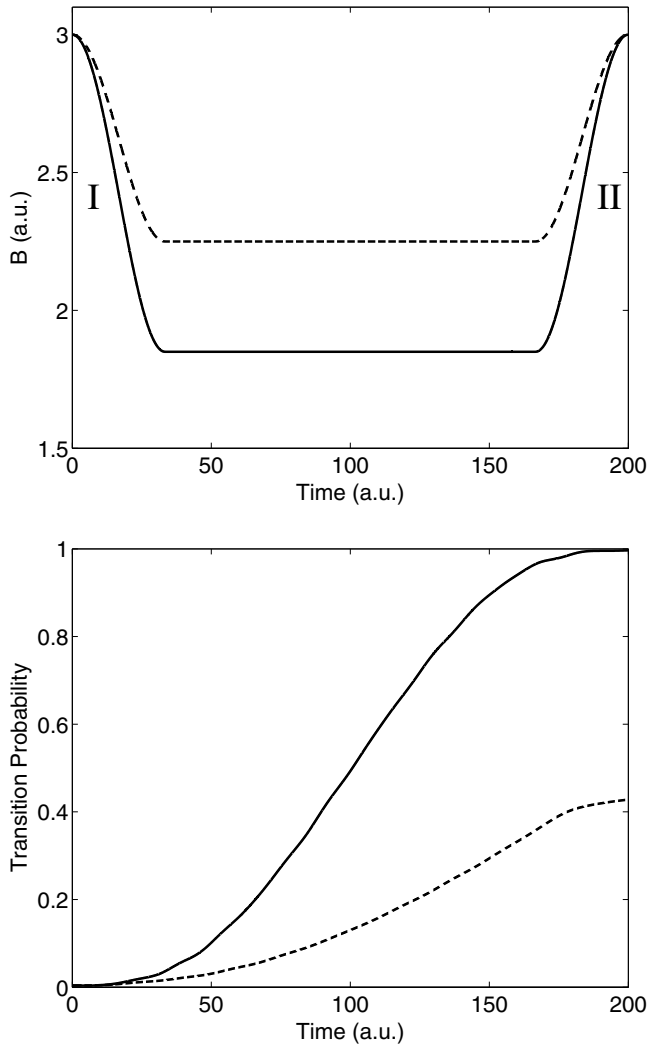

Figure 2. Upper panel: variation of the magnetic field with time. The change of the field at I and II corresponds to the transition arrows indicated in figure 1. Lower panel: the probability of finding the electron in the right well versus time. The solid (dashed) line shows the case where a complete (partial) transfer is achieved.

$t \sim 200$ au the magnetic field is turned on again to its initial value, and the electron is settled in the 'right' state, $\Psi_{\mathrm{R}} = \frac{1}{\sqrt{2}}(\psi_{\mathrm{g}} - \psi_{\mathrm{u}})$.

The upper panel of figure 2 shows how the magnetic field strength changes with time, corresponding to the field changes I and II in figure 1. The lower panel shows the probability of finding the electron in the right well versus time. The transition is seen to be complete after about 200 au, which corresponds to about 100 ps in a real system. Hence, the transition time is more than three orders of magnitude shorter than the coherence time reported in the experiment of Gorman *et al* [8]. We notice that the strength of the reversed magnetic field (i.e. the reduction of the static field) is of crucial importance to achieving a complete electron transfer. If the reduction of the magnetic field is not sufficient (upper panel of figure 2, dashed

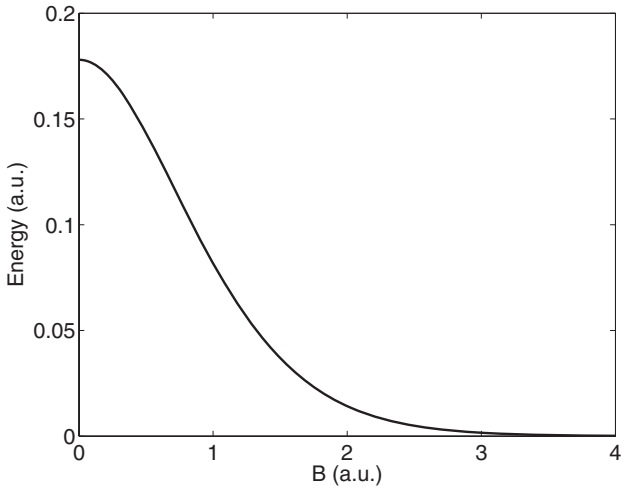

Figure 3. Energy difference, $\Delta E = \varepsilon_u - \varepsilon_g$, between the two lowermost energy states ψ_g and ψ_u in figure 1 (lower panel) as a function of the magnetic field strength.

line), only a partial transfer results (around 40% probability), as seen in the lower panel (dashed curve). However, also in this situation a complete transfer can be achieved by using a longer pulse. Assuming the pulse has the same form the transition time is about 400 au in this case.

For a given magnetic field strength B the time T it takes to obtain a complete population inversion between the Ψ_L and Ψ_R states is inversely proportional to the energy separation of the ψ_g and ψ_u states, i.e. [14]

$$T = \frac{\pi}{\Delta E}, \tag{13}$$

with $\Delta E(B) = \varepsilon_u(B) - \varepsilon_g(B)$, and ε_g and ε_u the energy of the *gerade* and *ungerade* molecular states, respectively. Figure 3 displays how the energy difference ΔE varies with the magnetic field strength. The relation (13) can be used to predict the pulse duration required to achieve complete charge transfer between the wells. However, the actual transition period will always be somewhat longer than this prediction because of the turn-off and turn-on times of the magnetic field, and equation (13) can be considered to be a lower bound. If the turn-off and turn-on times are short in comparison with the total pulse duration, then T should be a good estimate for the transition period. For the two examples given in figure 2, $T = 164$ and 372 au, respectively, which compare well to the 'true' values of 200 and 400 au.

In figure 4 we show in more detail how the electronic probability density changes in time during the switching. In the first snapshot the electron is in the initial localized state in the left well. In the next three snapshots the charge cloud is gradually transferred from the left to right well through delocalized states. The slight asymmetry in the y-direction seen in the middle panels arises from our choice of the initial state wavefunction. The last snapshot shows the electron as it has become fully transferred to the right-state and by inspection a complete (more than 99.9%) transition is achieved.

Summarizing, we have demonstrated the principle behind a fast magnetic switching that may transfer an electron between the two wells of a double quantum dot with 100% probability. The functioning of the switch is crucially dependent on properly adjusted field strengths and durations of the field pulses. Our model calculations demonstrate that such a transition can be achieved with an appropriate setup of the time-dependent magnetic field. In our model the

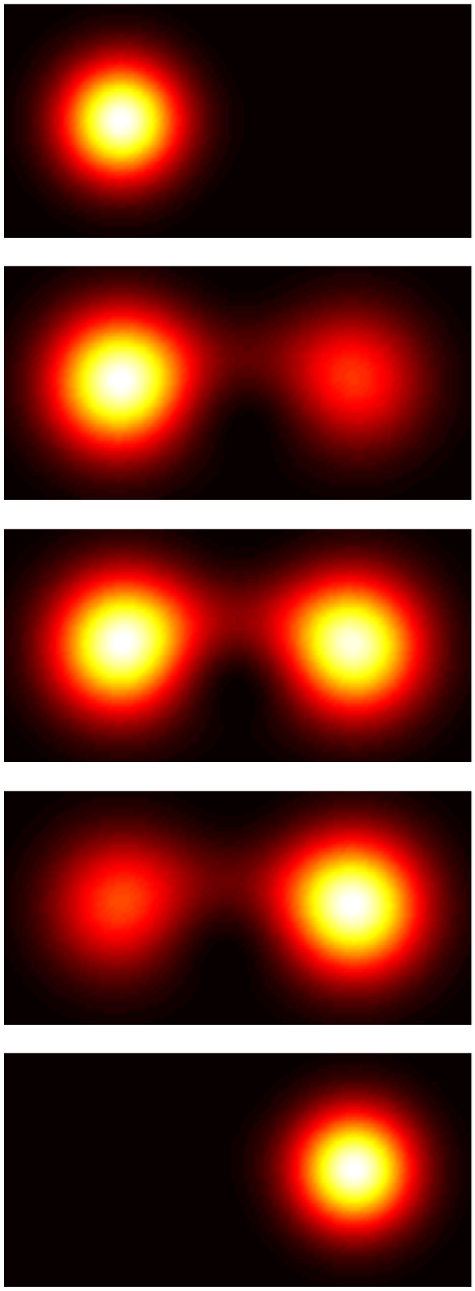

Figure 4. Snapshots of the electronic probability density for five different stages of the magnetic switching, $t = 0$, 80, 100, 120, and 200 au. The axis ranges are from $x = -3$ to 3 au (horizontal direction) and $y = -1.5$ to 1.5 au (vertical direction).

(This figure is in colour only in the electronic version)

transition is performed on the timescale of hundred picoseconds and with realizable magnetic field strengths. In practice such fast switching might not be achievable at present, but the model

calculations can be scaled to more favourable circumstances, e.g. by using larger distance between the two wells or by fabricating a bridge or neck between the two dots. Similar transitions in multi-electron double quantum dots may be achieved by balancing the strength and time dependence of the magnetic fields towards the particular electronic structure of the multi-electron double quantum dot. It is hoped that the presented physical model will motivate an experimental demonstration of controlled electron transfer by magnetic switching. The discussed controlled electron transfer mechanism in the double dot system can make it a suitable candidate for a logical gate with possible applications in quantum computation or other similar applications, as well as an element in studies of so-called artificial molecules.

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